ASSIGNMENT 1 ANSWERS

1)

n: # of possible patterns $(n=5)$

m: # of different length to produce $(m=3)$

b_i = # of pieces for length i $(i=1,\dots,m)$

a_{ij}: # of sheets of length i in the pattern j $(i=1,\dots,m\ ;\ j=1,\dots,n)$

X_j = # of sheets cut using pattern j

Length \ Pattern	1	2	3	4	5
3 ft	3	2	1	0	1
4 ft	0	1	2	1	0
6 ft	0	0	0	1	1
Total	9	10	11	10	9

$$\text{Min} \sum_{j=1}^{n} X_j$$

s.t.

$$\sum_{j=1}^{n} a_{ij} X_j \geq b_i \quad \forall i=1,\dots,m$$

$$X_j \geq 0 \text{ , integer} \quad \forall j=1,\dots,n$$

2) $(X_i, Y_i) \rightarrow$ Coordinates of the new machine i $(i=1,2)$

$$(P_1) \quad \text{Min} \quad z = |X_1-17| + |X_1-65| + |X_1-95| + |Y_1-15| + |Y_1-52| + |Y_1-5|$$
$$+ |X_2-17| + |X_2-65| + |X_2-95| + |Y_2-15| + |Y_2-52| + |Y_2-5|$$

s.t.

$X_1 \leq 100$ $\quad$ $Y_1 \leq 70$

$X_2 \leq 100$ $\quad$ $Y_2 \leq 70$

$|X_1 - X_2| \geq 8$ $\quad$ $|Y_1 - Y_2| \geq 8$

$X_1, X_2, Y_1, Y_2 \geq 0$

To linearize the (P_1) formulation, the following variables are defined:

$\alpha_1 = |X_1-17|$ $\quad$ $\alpha_4 = |X_2-17|$ $\quad$ $\beta_1 = |Y_1-15|$ $\quad$ $\beta_4 = |Y_2-15|$

$\alpha_2 = |X_1-65|$ $\quad$ $\alpha_5 = |X_2-65|$ $\quad$ $\beta_2 = |Y_1-52|$ $\quad$ $\beta_5 = |Y_2-52|$

$\alpha_3 = |X_1-95|$ $\quad$ $\alpha_6 = |X_2-95|$ $\quad$ $\beta_3 = |Y_1-5|$ $\quad$ $\beta_6 = |Y_2-5|$

The formulation is updated such that:

$$(P2) \quad \text{Min} \quad z = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$$

s.t.

$\alpha_1 \geq X_1-17$, $\alpha_1 \geq -(X_1-17)$ $\quad$ $\beta_1 \geq (Y_1-15)$, $\beta_1 \geq -(Y_1-15)$

$\alpha_2 \geq X_1-65$, $\alpha_2 \geq -(X_1-65)$ $\quad$ $\beta_2 \geq Y_1-52$, $\beta_2 \geq -(Y_1-52)$

$\alpha_3 \geq X_1-95$, $\alpha_3 \geq -(X_1-95)$ $\quad$ $\beta_3 \geq Y_1-5$ $\quad$ $\beta_3 \geq -(Y_1-5)$

$\alpha_4 \geq X_2-17$, $\alpha_4 \geq -(X_2-17)$ $\quad$ $\beta_4 \geq Y_2-15$ $\quad$ $\beta_4 \geq -(Y_2-15)$

$\alpha_5 \geq X_2-65$, $\alpha_5 \geq -(X_2-65)$ $\quad$ $\beta_5 \geq Y_2-52$ $\quad$ $\beta_5 \geq -(Y_2-52)$

$\alpha_6 \geq X_2-95$, $\alpha_6 \geq -(X_2-95)$ $\quad$ $\beta_6 \geq Y_2-5$ $\quad$ $\beta_6 \geq -(Y_2-5)$

$X_1 - X_2 \geq 8$ $\quad$ $Y_1 - Y_2 \geq 8$

$-(X_1 - X_2) \geq 8$ $\quad$ $-(Y_1 - Y_2) \geq 8$

$X_1 \leq 100$ $\quad$ $Y_1 \leq 70$

$X_2 \leq 100$ $\quad$ $Y_2 \leq 70$

$X_1, X_2, Y_1, Y_2 \geq 0$

$\alpha_j, \beta_j \geq 0 \quad \forall j = (1, .., 6)$

3. Static Workforce scheduling Problem

	Day		Requirement
Monday	Day	1	17
Tues.	"	2	13
Wed.	"	3	15
Thurs.	"	4	19
Friday	"	5	14
Sat.	"	6	16
Sunday	"	7	11

Workers take off two days; either Sat. and Sun. or any two week days.

X_i = sequence of 5 days an employee on schedule i works.

one schedule works M-F w/ sat. + Sun off.

Other schedule combinations are ${}_5C_2 = 10$

Total of ~~#~~ 11 ways to schedule

So X_i, $i = 1, \dots, 11$

~~Minimize~~

R = Thursday : Schedules

	Days off		Work days
X_1	S, S	→	M - F
X_2	M, T	→	W - Sun
X_3	M, W	→	R - Sun, T
X_4	M, R	→	T, W, F, S, Sun
X_5	M, F	→	T, W, R, S, S
X_6	T, W	→	R, F, S, S, M
X_7	T, R	→	W, F, S, S, M
X_8	T, F	→	W, R, S, S, M
X_9	W, R	→	F, ~~S~~S, M, T
X_{10}	W, F	→	R, S, S, M, T
X_{11}	R, F	→	S, S, M, T, W

⇓

minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$

$$X_1 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} \geq 17$$
$$X_1 + X_3 + X_4 + X_5 + X_9 + X_{10} + X_{11} \geq 13$$
$$X_1 + X_2 + X_4 + X_5 + X_7 + X_8 + X_{11} \geq 15$$
$$X_1 + X_2 + X_3 + X_5 + X_6 + X_8 + X_{10} \geq 19$$
$$X_1 + X_2 + X_3 + X_4 + X_6 + X_7 + X_9 \geq 14$$
$$X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} \geq 16$$
$$X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} \geq 11$$

X_i integer $i = 1, \dots, 11$

20/20

A4. Let $x_i = 1$ if player i starts
$x_i = 0$ otherwise
Then appropriate IP is

$$\max z = 3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + x_7$$
s.t. $x_1 + x_3 + x_5 + x_7 \geq 4$ (guards)
$x_3 + x_4 + x_5 + x_6 + x_7 \geq 2$ (forwards)
$x_2 + x_4 + x_6 \geq 1$ (center)
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5$
$3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7 \geq 10$ (BH)
$3x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 + x_6 + 2x_7 \geq 10$ (SH)
$x_1 + 3x_2 + 2x_3 + 3x_4 + 3x_5 + 2x_6 + 2x_7 \geq 10$ (REB)
$x_6 + x_3 \leq 1$
$-x_4 - x_5 + 2 \leq 2y$
(If $x_1 > 0$ then $x_4 + x_5 \geq 2$)
$x_1 \leq 2(1-y)$
$x_2 + x_3 \geq 1$
$x_1, x_2, \ldots x_7, y$ are all 0-1 variables

A5.
Let x_i = Number of workers employed on Line i
$y_i = 1$ if Line i is used, $y_i = 0$ otherwise
$$\min z = 1000y_1 + 2000y_2 + 500x_1 + 900x_2$$
s.t. $20x_1 + 50x_2 \geq 120$
$30x_1 + 35x_2 \geq 150$
$40x_1 + 45x_2 \geq 200$
$x_1 \leq 7y_1$
$x_2 \leq 7y_2$
$x_1 \geq 0$, $x_2 \geq 0$, $y_1 = 0$ or 1, $y_2 = 0$ or 1

A6.
a. Let $x_i = 1$ if disk i is used, $x_i = 0$ otherwise
$$\min z = 3x_1 + 5x_2 + x_3 + 2x_4 + x_5 + 4x_6 + 3x_7 + x_8 + 2x_9 + 2x_{10}$$
s.t. $x_1 + x_2 + x_4 + x_5 + x_8 + x_9 \geq 1$ (File 1)
$x_1 + x_3 \geq 1$ (File 2)
$x_2 + x_5 + x_7 + x_{10} \geq 1$ (File 3)
$x_3 + x_6 + x_8 \geq 1$ (File 4)
$x_1 + x_2 + x_4 + x_6 + x_7 + x_9 + x_{10} \geq 1$ (File 5)
$x_i = 0$ or 1 $(i=1,2,\ldots 10)$

b. If $x_3 + x_5 > 0$, then $x_2 \geq 1$ yields
$1 - x_2 \leq 2y$
$x_3 + x_5 \leq 2(1 - y)$ $y=0$ or 1
(need M=2 because $x_3 + x_5 = 2$ is possible)

10|10

7. Graphical Method

MINIMIZE $Z = -3x_1 + 12x_2$

s.t. $3x_1 - x_2 \le 3$ (1)

$-x_1 + 4x_2 \le 8$ (2)

x_1 & $x_2 \ge 0$

$3x_1 - \frac{27}{11} = 3$

$33x_1 - 27 = 33$

$33x_1 = 60$ $x_1 = \frac{60}{33} = \frac{20}{11}$

$3x_1 - x_2 = 3$

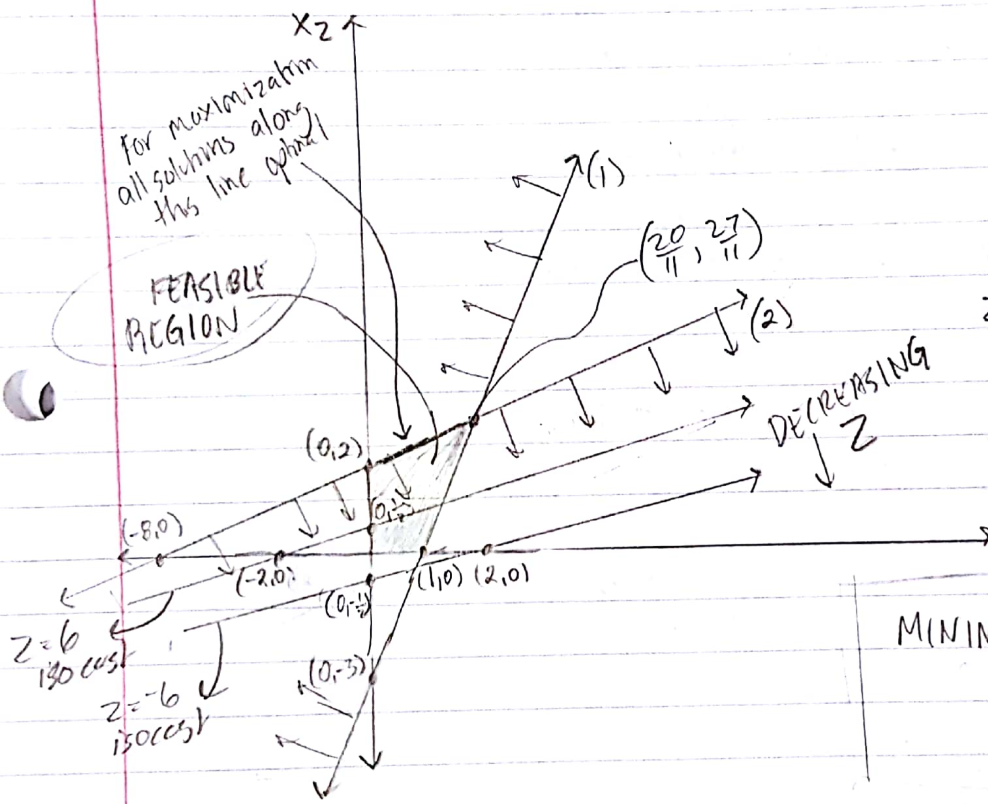
$(-x_1 + 4x_2 = 8) \cdot 3$

$-3x_1 + 12x_2 = 24$

$11x_2 = 27$

$x_2 = 27/11$

$x_1 = 20/11$



$Z = 6 \Rightarrow (0, \frac{1}{2})\ (-2, 0)$

$Z = -6 \Rightarrow (0, -\frac{1}{2})\ (2, 0)$

MINIMIZE $Z = -3x_1 + 12x_2$

$(x_1^*, x_2^*) = (1, 0)$ ✓

$Z^* = -3$

MAXIMIZE $Z = -3x_1 + 12x_2$

Infinite Optimal Solutions

$x_1 = 0a + \frac{20}{11}(1-a) \Rightarrow \frac{20}{11} - \frac{20}{11}a$

$x_2 = 2a + \frac{27}{11}(1-a) \Rightarrow \frac{27}{11} - \frac{5}{11}a$

$(x_1^*, x_2^*) = \left(\frac{20}{11} - \frac{20}{11}a, \frac{27}{11} - \frac{5}{11}a\right)$ for $0 \le a \le 1$

$Z^* = 24$

✓