Modeling the costs and benefits of delayed product differentiation


— Increased variety of products
  ⇒ Large safety stock requirements.

— Delayed product differentiation is a solution to reduce safety stocks.
  ⇒ Process will not commit the WIP into a particular product until a later point.

Product A
  0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0

Product B
  0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0

⇒ Delaying differentiation point through product/process redesign.

- Component standardization
- Modular design
- Process restructuring
The Model:

Consider a production system that produces two end products:

- Two end products.
- In total, \( N \) stages.
- \( k \) of the stages are common operations.
- \( N-k \) distinct operations.

\( 0 \leq k \leq N-1 \)

\( D_i(t) \) : demand for product \( i \), in period \( t \), normally distributed i.i.d. for \( i = 1, 2 \).

\[ \mathbb{E}(D_i(t)) = \mu_i \] \[ \text{Var}(D_i(t)) = \sigma_i^2 \]

\( \text{cov}(D_1(t), D_2(t)) = \rho \sigma_1 \sigma_2 \) \( (\rho: \text{correlation coefficient}) \)

\[ \sigma_{12} = \sqrt{\text{Var}(D_1(t) + D_2(t))} = \sqrt{\sigma_1^2 + 2 \rho \sigma_1 \sigma_2 + \sigma_2^2} \]
\[ S_{12} \leq S_{1} + S_{2} \text{ for any } S_{1}, S_{2}, \text{ and } S_{12} \text{ decreases as } S \text{ decreases.} \]

Developing a simple cost function for a system where operation \( k \) is the last common operation:

- Assume that sufficient safety stock is held at each buffer so that we can analyze each stage individually: (decoupled, high service levels)
- Each stage uses an order-up-to level policy
- Service levels for different buffers are the same (could be relaxed)
- Quality of the product does not depend on the point of differentiation.

**Notation:**

\( S_i \): Average investment cost per period if operation \( i \) became a common operation. (\( S_i \) could be positive or negative)
\( \lambda_{i}(k) \): lead time for operation \( i \) when operation \( k \) is the last common operation

\( \rho_{i}(k) \): Processing cost per unit for operation \( i \)

\( h_{i}(k) \): Inventory holding cost for holding one unit of inventory at buffer \( i \)

\( 2 \): safety factor associated with target level for each buffer

For each buffer facing a demand with mean \( \mu \) and standard deviation \( \sigma \) (following an order-up-to policy with review period of 1)

- Average on order (WIP or in-transit) inventory
  \[ = \frac{\mu}{2} + 2\sigma \sqrt{2} \frac{\sigma}{\mu + 1} \]
$Z(k)$: Total relevant cost when the last common stage is $k$. (per period)

$$Z(k) = \sum_{i=1}^{k} c_i + \sum_{i=1}^{N} p_i(k)(m_i + m_2) + \sum_{i=1}^{N} h_i(k)(N_i(k)/m_1 + m_2)$$

$$\quad + \sum_{i=1}^{k} h_i(k) \left[ \frac{(m_i + m_2)}{2} + 2\sigma_12 \sqrt{N_i(k)+1} \right]$$

$$\quad + \sum_{i=k+1}^{N} h_i(k) \left[ \frac{(m_i + m_2)}{2} + 2\sigma_12 \sqrt{N_i(k)+1} \right]$$

1. Total average investment cost per period
2. Total processing cost
3. Total WIP cost
4. Buffer inventory cost for common operation
5. Buffer inventory cost for distinct operation
Optimal start overrun operation

\[ k^* = \arg \min_k \{ \frac{8}{k} \} \quad 0 \leq k \leq N - 1 \]

- Example: Standardization of Components

- Manufacturing two types of printers: mono and multi-color.

- Three major steps:
  - Printed circuit board assembly (PCB)
  - Final assembly and test (FAST)
  - Final customization (Customization)

- Negative correlation of the demands for two products.

- \( N = 3 \), \( k = 0 \) when all operations are distinct.

- For delayed differentiation we can standardize either 1st step or both first and second steps.
Standardizing: PCA is difficult, requires investment
require standardizing higher unit cost.
"head drive"

Standardizing FAT: easy, no high investment,
require standardizing similar unit cost as the distinct
"print mechanics" process

$s_1 \gg s_2$ \quad \forall i(k) = \forall i$ for each $i$

$p_i(k) = p_i$ for all $i$ when $k = 0$

$p_i(k) = p_i + p_{i-1}$ if $i \leq k$ when $k \geq 1$

$p_i(k) = p_i$ if $i > k$ when $k \geq 1$

$\beta_i \geq 0$ additional material + processing cost
when operation $i$ is standardized

$\beta_1 > \beta_2 \geq 0$ in our example.

$h_i(k) = b_i$ for all $i$ when $k = 0$ (no standardization)

$h_i(k) = b_i + (s_1 + s_2 + \ldots + s_k)$ for $i \leq k$ when $k \geq 1$

$h_i(k) = b_i + (s_1 + s_2 + s_k)$ if $i > k$ when $k \geq 1$

$s_i$: additional value added when stage $i$ is standardized
in our example: $s_1 > 0$ and $s_2 > 0$

- we can evaluate $z(0)$, $z(1)$, and $z(2)$.
- Comparing $z(1)$ and $z(0)$.

\[
\begin{align*}
\hat{z}(1) - \hat{z}(0) &= S_1 + \beta_1 (M_1 + M_2) + 3 S_1 \left( \frac{M_1 + M_2}{2} \right) \\
&\quad + \sum_{i=1}^{3} \beta_i (M_i + M_2) \left[ \sqrt{N_i + 1} + \sqrt{N_3 + 1} \right] \\
&\quad + 2 \left( \sqrt{N_i + 1} \right) \left( (b_i + S_1) \sqrt{1_2} - b_1 (S_i + T_2) \right).
\end{align*}
\]

5) added safety stock cost at buffer 2 and 3.
6) saving in $z(1) - z(0)$ at buffer 1.

Since $S_1$ and $S_2$ is large, we expect $\hat{z}(1) > \hat{z}(0)$ = standardization at only stage 1 does not bring net benefit.

\(\uparrow\)

Not expected profit.
So: $k^* = 0$ or $k^* = 2$ (No standardization or standardize both stage 1 and two) should be considered

(Example: Modular design)

- Dishwasher manufacturer. manufactured in two colors: white and black
- Manufacturing steps:
  - Fabrication + Integration + shipping = Distribution
- Currently metal frames with different colors are added at second stage:

```
  O --> □ --> O --> □ --> Device A
  \        \                 \\
  □ --> □                   □ --> □ --> Device B
  \        Bundling + Shipping
  □ --> Device B
```

$N = 3$, $k = 1$

- Consider modular design for metal frames;
  - Generic metal frame + plastic panel that specifies the color.
- With this modular design, generic dishwashers will be sent to dist. center, and dish center will add the plastic-colored panels.

Thus $k = 2$ in this case.

- When $k = 1$, $\lambda_i(1) = \lambda_i$, $\rho_i(1) = \rho_i$, $\mu_i(1) = \mu_i$ for each $i$.

And $n_2 \gg n_3$ since stage 2 has assembly + shipping.

- When $k = 2$, $-S_2$ investment cost for the modular design.

- No significant change in the first two stages.

- Cost parameters at the third stage change:

  $\lambda_3(2) = \lambda_3 + \alpha$, $\rho_3(2) = \rho_3 + \beta$

  $\mu_3(2) = \mu_3 + \delta$

$\zeta(2) - \zeta(1) = S_2 + \beta (M_1 + M_2) + \delta (M_1 + M_2) / 2$

$$+ b_2 \left[ \alpha_1 - (\alpha_1 + \alpha_2) \right] \sqrt{M_2 + 1} + \alpha (\alpha_1 + \alpha_2) / 2$$

$\geq \alpha_1 \text{ 2nd stage}$

$$\times \left[ (b_2 + \delta) \sqrt{M_3 + \delta_1} - b_3 \sqrt{M_3 + \delta_1} / \sqrt{M_3 + \delta_1} \right]$$
In this example, \( \alpha = 0, \beta > 0, \delta > 0 \) and \( \varepsilon_2 \) is small and \( \varepsilon_1 \) is large, therefore it is likely that 

\[ f(2) < f(1) \]

**Example:** Process restructuring: Reversal of operations

- Conversion example: Red and Blue sweater
  - Sweater production
    - dyeing \( \rightarrow \) knitting \( \rightarrow \) distribution
  - Processing time of dyeing is much shorter.
- Currently \( k = 0 \), \( N = 3 \)

\[ 0 \rightarrow \bigtriangleup \rightarrow 0 \rightarrow \bigtriangleup \rightarrow \text{Red} \quad k = 1. \]

\[ 0 \rightarrow \bigtriangleup \rightarrow 0 \rightarrow \bigtriangleup \rightarrow 0 \rightarrow \bigtriangleup \rightarrow \text{Blue}. \]

- When \( k = 0 \), \( \nu_1(0) = \nu_1 \); \( p_1(0) = 0 \); \( h_1(0) = b + d_1 \)

\[ h_2(0) = b + d_1 + d_2 \quad h_3(0) = b + d_1 + d_2 + d_3 \]

\( m_1 < n_2 \), \( \delta_1 > \delta_2 \) (dyeing adds more value, expensive dyeing machine)
Consider the case of dyeing and knitting, reversed:

\[ \begin{align*}
N_1(1) &= N_2, \\
N_2(1) &= N_1, \\
P_1(1) &= P_2, \\
P_2(1) &= P_1,
\end{align*} \]

\[ \begin{align*}
h_1(1) &= b + \delta_2, \\
h_2(1) &= b + \delta_2 + \delta, \\
h_3(1) &= b + \delta_1 + \delta_2 + \delta_3
\end{align*} \]

\[
Z(0) - Z(0) = S_1 + \left( \delta_2 N_1 - \delta_1 N_2 \right) (m_1 + m_2) + \left( \delta_2 - \delta_1 \right) (m_1 + m_2) / 2
\]

\[
+ \frac{1}{2} \left[ (b + \delta_2) \sqrt{\bar{N}_2 + 1} - (b + \delta_3) \sqrt{\bar{N}_1 + 1} \right]
\]

\[
+ \frac{1}{2} \left( b + \delta_1 + \delta_2 \right) \left( \sqrt{\bar{N}_1 + 1} + \sqrt{\bar{N}_1 + 1} \right)
\]

safety stock reduction after first stage

safety stock reduction after second stage

\[
Z(1) - Z(0) = S_1 + \left( \delta_2 N_1 - \delta_1 N_2 \right) (m_1 + m_2)
\]

\[
+ \frac{1}{2} \left( \delta_2 \left( \sqrt{\bar{N}_1 + 1} + \sqrt{\bar{N}_2 + 1} \right) + \left( \delta_2 - \delta_1 \right) (m_1 + m_2) \right) / 2
\]

\[
+ \frac{1}{2} \left[ (b + \delta_2) \sqrt{\bar{N}_2 + 1} - (b + \delta_1 + \delta_2) \sqrt{\bar{N}_1 + 1} \right]
\]

\[
\left( \delta_2 \leq \delta_1, \quad \delta_2 \leq \delta_1 \right)
\]

\[
\times \to \text{savings: increase as } \bar{N}_2 \text{ or as } \delta_1 - \delta_2
\]

\[
\Rightarrow \text{ reverse short ops at early stage with longer operation at later stage, or high value-added operation early stage with low value-added operation at later stage}
\]